

Statistics of continuous motion force events in a driven 2D granular array

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A quasi-two dimensional array of cylinders was pushed along a horizontal aluminum track at constant velocity and the total driving force was sampled at 10 KHz. Operating in a continuous motion regime, we found driving force fluctuations that follow a power law distribution with a sharp upper cutoff, a behavior which persisted even while varying the number of cylinders, the geometry and the driving velocity. We also found that while the total force decreases with velocity, the maximum event size increases with velocity, a feature which we interpret as evidence of the system of contacts organizing themselves into a self-organized critical state.

Stick-slip behavior in granular materials has been a topic of intense interest in recent years[1–6]. It is of vital importance in understanding avalanches [7], earthquake dynamics [8], and the dynamics of solid on solid friction [9, 10]. The models used in the experimental and theoretical studies of such behavior typically involve pushing a block, an array of particles, or a plate on top of an array of particles with a spring. In each regime, potential energy is stored in the pushing spring and between individual contacts or grains until the intergrain forces reach a maximum, and the whole array slips. As the stiffness of the pushing spring is increased (or the driving speed is increased sufficiently), the driving force eventually changes from a series of linear rising and falling stick-slip events to a continuous motion regime characterized by a very irregular force signal[1, 3, 9, 11].

In a simplified model of a granular material, Radjai *et al*[1] performed a stick slip experiment on a one-dimensional array of cylinders. They pushed their array along a horizontal track with a relatively weak spring while measuring the driving force and obtained a series of clear stick-slip events whose statistics they studied in some detail. However, they did not study this system in the continuous motion regime. Our experiment improves the sensitivity (by 70 times) and significantly extends the work of Radjai *et al*[1, 6] to the regime of two dimensional continuous motion. In addition, we provide some insight into the nature of the continuous motion regime force fluctuations that have up to this point (see [3]) remained unstudied in other model granular systems.

Our experimental setup uses a very heavy ($\approx 250kg$) 1.8 meter lathe bed with a kinematically mounted carriage and is shown in Figure 1. The carriage is driven by a 40 TPI lead screw which is driven from the rear by a 1/5 hp gearmotor, which is further geared down by use of a 20 tooth worm gear. This provides an extremely robust drive that by calculation, can exert a maximum of 1000 pounds of force to the carriage, and allows us to ignore loading due to the cylinders. Attached to the forward face of the carriage is a vertical set of linear motion rails which hold a 1 inch thick Al plate; this allows us

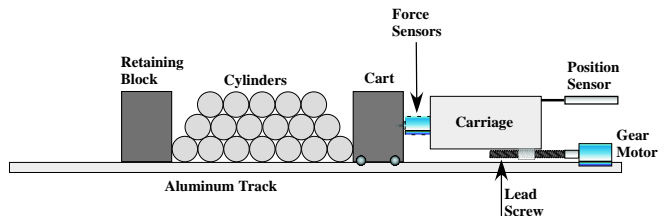


FIG. 1: Schematic of our apparatus.

to adjust the vertical position of two 10 N plunger type strain gauge sensors which are mounted to this plate. These sensors push symmetrically on a low friction cart (mass 3.526 ± 0.001 kg). We stack 7.62 cm long aluminum cylinders (diameter 2.54 cm, mass 0.1044 ± 0.0001 kg) in a hexagonally packed array (as shown in Figure 1) and prevent them from rolling away by use of a retaining block whose mass varies depending on the size of the array used. These cylinders and the retaining block all rest on an aluminum plate which is rigidly screwed to the lathe bed; the cart rides on ball-bearing wheels which rest on the flat rails of the lathe. We record the position of the carriage and hence the cart through use of a resistive position sensor.

We wrote customized routines to control the entire experiment from a PC running LabView 6i; all data i/o is routed through a National Instruments 16 bit A/D board with a maximum sampling rate of 333 kS/s for a single channel. In practice, since we sample 4 channels, we are limited to 83 kS/s per channel. Repeated tests reveal the typical time scale for force fluctuations is about 1 ms, so we oversample at 10 kHz, far below our maximum available rate. The force resolution was experimentally determined as 0.5 mN, about 3 times larger than the 0.15 mN, one would expect from the 16 bit resolution of the A/D board; this measured value agrees with the manufacturer's specifications, and is 70 times better than that reported by Radjai *et al*[1, 6]. Our position sensor resolution was measured to be 10 μ m.

The cylinders and the track upon which they move are

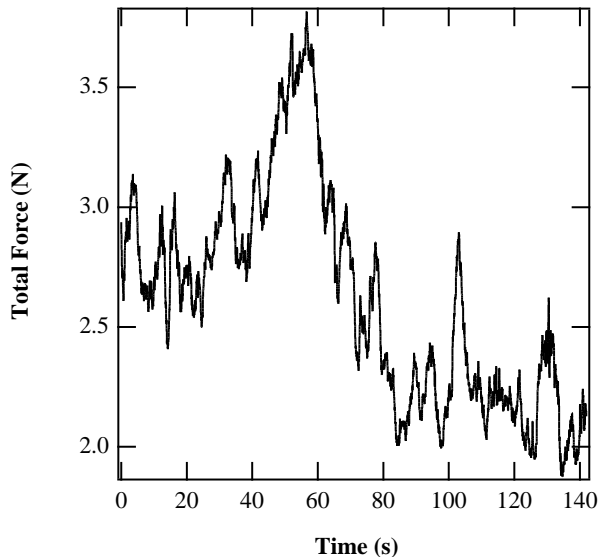


FIG. 2: The force versus time curve for a typical data run. On this time scale, the smoothed data is indistinguishable from the raw data.

washed with acetone prior to the start of each run. Then, each data run starts by positioning the array at the same initial position on the aluminum plate (to within $25 \mu\text{m}$). We record the force from each sensor, and the position of the cart, all at a 10 KHz sampling rate.

For each run, we compute the total force (see Figure 2) and apply 40 pass binomial smoothing to this raw data. We chose the number of passes of smoothing to reduce the digital noise present in the signal without significantly altering the fluctuations in the force signal. Figure 3 shows an enlargement of a the data shown in Figure 2 along with the smoothed representation of this data. It is important to note that the amount of smoothing to use is inherently subjective; however, once we settled on an amount of smoothing, we stuck with this amount for all subsequent runs.

We are interested in the nature of the force fluctuations, so we implemented an algorithm to search for these events in our smoothed data. Each run was broken into a series of continuously increasing (stick) or continuously decreasing (slip) force events, and the size of these events as well as their duration were recorded.

We investigated the statistics of the force events under many different conditions within the continuous motion regime. Since the force sensors we have are very rigid (resonant frequency = 300 kHz, max deflection = 0.28mm), we do not see the classic stick-slip events seen by other groups [1, 3] unless we insert a weak spring between the force sensors and the pushing cart. Running in this continuous motion regime, our force signal is much more erratic in time, and does not resemble the classic stick-slip event structure. However, for convenience, we

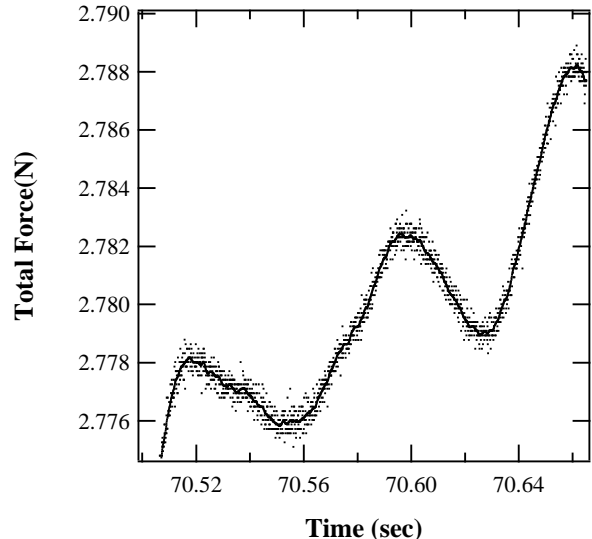


FIG. 3: A 14 ms sample of the data shown in Figure 2 with the raw data and the smoothed representation of that data.

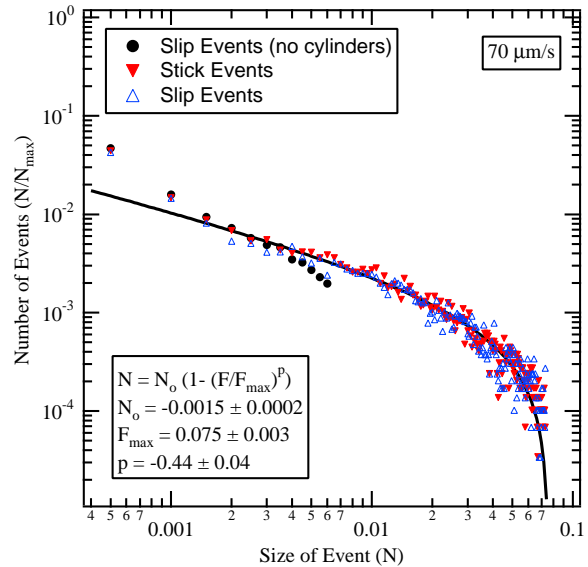


FIG. 4: Distribution of event sizes for a typical run. This data set resulted from a 22 element hexagonal pile of Aluminum cylinders pushed at $70 \mu\text{m/s}$. The stick and slip events are both shown, as well as as the data from a $70 \mu\text{m/s}$ run pushing only the retaining block and the pushing cart.

adopt the same language; but in our case stick and slip events refer to a string of continuously rising or falling force values, respectively.

To test the effect of drive speed on the force fluctuations, we conducted a series of runs at speeds from 10 to $120 \mu\text{m/s}$, all involving a hexagonal stacking of 22 aluminum cylinders; seven on the bottom layer, followed by a layer of six, then five and finally 4 on the top layer. The confining block had a mass of 594 g, which was necessary

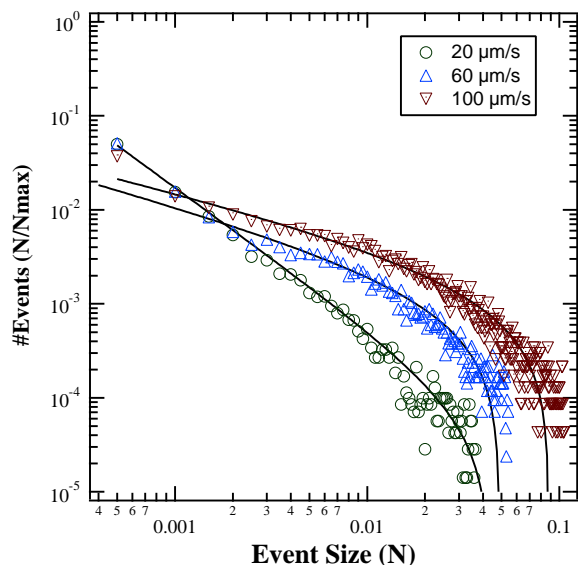


FIG. 5: Distribution of slip event sizes for several different driving speeds.

to stop the pile of cylinders from collapsing. Figure 4 shows the distribution of event sizes at $70 \mu\text{m/s}$. Also shown in this plot (solid circles) is the distribution of slip events when we push the retaining block and pushing cart only. This strongly suggests that the leftmost six points in Figure 4 correspond to the small and numerous events due primarily to the retaining block and that the addition of the cylinders is responsible for the 10 fold increase in the maximum event size at this run speed. The stick and slip events follow essentially the same distribution, a result very different from the behavior seen by Radjai and Nasuno [1, 3].

Figure 5 shows the distribution of event sizes at 20, 60, and $100 \mu\text{m/s}$ (the others were omitted for clarity). At run speeds of 10 and $20 \mu\text{m/s}$, the data more closely follows a pure power law, whereas the higher speed runs show a deviation from this behavior, with a sharp upper cutoff on the maximum event size. These events (in Figures 4 and 5) are well modeled by a power law of the form

$$N = N_0 \left[1 - \left(\frac{F}{F_{max}} \right)^p \right], \quad (1)$$

where F_{max} is a maximum event size. Figure 6 shows that this maximum event size increases with driving speed in a non-linear fashion. A striking feature of the data is that while the event sizes (above $10 \mu\text{m/s}$) increase with speed, the mean pushing force decreases with speed as shown in Figure 6. The decrease in the mean force with speed is in accord with Burridge-Knopoff models of friction [12]. The increase in event sizes with speed *may* be understandable if we think of the frictional contacts withing the cylindrical array as self-organized into

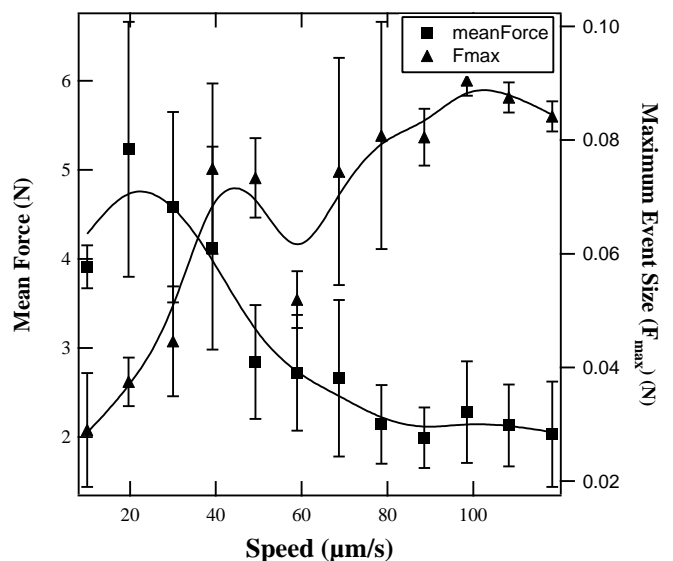


FIG. 6: Mean force and Maximum event size for different driving speeds. The error bars represent one standard deviation errors, and the splined fits are only meant to guide the eye.

a critical state; at higher speeds there is more kinetic energy available to turn what would be a small event into a larger one. This behavior is consistent with depinning models (see [13]) as well as dynamical models where energy is stored in elastic deformations of surface asperities [14].

Accepting that our cylinders have organized into a critical state would also imply that larger arrays would produce larger events by virtue of having more contacts. We tested this prediction by looking at two runs differing only in the number of cylinders. We looked at runs with 24 and 90 steel cylinders and found that the maximum event size increased roughly in proportion to the number of cylinders as shown in Figure 7.

We checked to see if the increase in event size was due to motor vibration by performing several different speed runs with two differently geared motors. One motor operated at high internal speed (and therefore vibrated more) and the other operated at low internal speed (producing the same drive speed, but less vibration). Three different speed runs at 10, 20, and $25 \mu\text{m/s}$ showed no striking differences, indicating that the motor vibration is not the cause of the striking cutoff evident in Figures 4 and 5. We do find that motor vibration becomes more prominent at speeds over $110 \mu\text{m/s}$, so we cannot extend our work in this range without a redesign of our apparatus.

The dependence of the event sizes on geometry and retaining mass was also explored. We explored 22 particle arrays with one to 5 layers and found no noticeable difference in the distribution of event sizes. In addition, increasing the retaining block mass appears to leave force fluctuations unaffected. However, as the retaining block

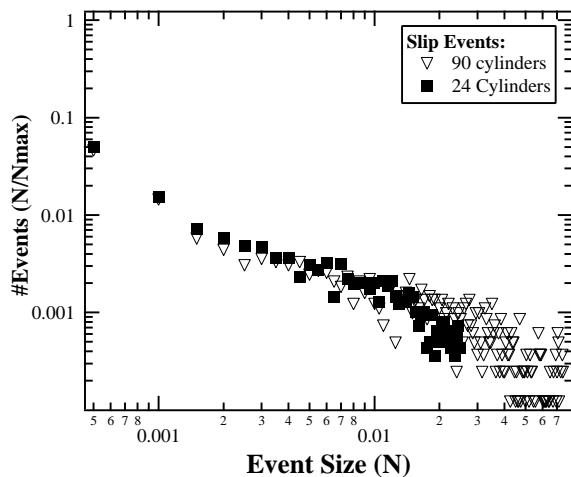


FIG. 7: Distribution of event sizes for a two runs differing only in the number of cylinders. Larger numbers of cylinders produce larger maximum event sizes.

mass is increased the rotations of many of the individual cylinders becomes frustrated. In a paper to be published elsewhere, we report on the correlations between cylinder rotations, total force, dilatancy, and collapse of the array.

In summary, we have investigated the behavior of a two dimensional array of cylinders pushed at constant velocity in the continuous motion regime. The primary behavior of this system is to exhibit force fluctuations that follow a power law with a finite size cutoff. The detailed shape of this distribution depends strongly on the pushing velocity, but not significantly on the geometry or amount of confining mass. Furthermore, although the absolute values of the fit parameters depend upon how much smoothing one applies to the raw data, the qualitative features of equation 1 do not depend on this choice. Our data implies that the distribution of event

sizes is potentially the result of the system reaching a self-organized critical state similar to avalanche and solid friction models.

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